SOLUTION OF SHORT QUESTIONS

Short Questions

Write the short answers of the following Questions:

Q.1: Define degree and radian measures.

Ans. **Degree:** If a circle is divided into 360º equal parts, then angle subtended by one part at the center of the circle is called a degree.

Radian: Radian is the measure of the angle subtended at the center of the circle by an arc, whose length is equal to the radius of the circle.

Q.2: Convert into radian measure:

[a] $120°$ (IA-2018)

Sol. $120° = 120 \times \frac{\pi}{180} = \frac{2\pi}{3} = \boxed{2.09\,\text{rad}}$

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[b] $22\frac{1}{2}°$ (IA-2016)

Sol. $22\frac{1}{2}° = 22.5° = 22.5 \times \frac{\pi}{180} = \boxed{0.39\,\text{rad}}$

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[c] $12°\ 40'$

Sol. $12°\ 40' = \left(12 + \frac{40}{60}\right)^0 = \left(12 + \frac{2}{3}\right)^0 = (12 + 0.6667)°$

$= 12.6667° = 12.6667 \times \frac{\pi}{180} = \boxed{0.22\,\text{rad}}$

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[d] $42°36'12''$ (IA-2017), (IA-2019)

Sol. Same as Q.1(iii) of Ex# 3.1 (see page # 116)

Q.3: Convert into degree measure:

[a] $\frac{\pi}{2}\,\text{rad}$

Sol. $\frac{\pi}{2}\,\text{rad} = \frac{\pi}{2} \times \frac{180}{\pi} = \boxed{90°}$

SOLUTION OF SHORT QUESTIONS

[b] 0.726 rad (IIA-2016), (IIA-2020)

Sol. $0.726 \text{ rad} = 0.726 \times \frac{180}{\pi} = \boxed{41°35'48''}$

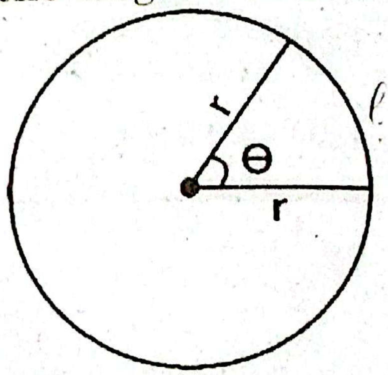
[c] $\frac{2\pi}{3}$ rad (IIA-2018)

Sol. $\frac{2\pi}{3} \text{ rad} = \frac{2\pi}{3} \times \frac{180}{\pi} \text{ rad} = \boxed{120°}$

Q.4: **Prove that $\ell = r\theta$.** (IIA-2016)

Sol. The ratio of 'ℓ' to the circumference $2\pi r$ of the circle is same as ratio of the angle θ to 2π.

$\ell : 2\pi r = \theta : 2\pi$



$$\frac{\ell}{2\pi r} = \frac{\theta}{2\pi}$$

$$\ell = \frac{\theta(2\pi r)}{2\pi}$$

$\ell = r\theta$ **Proved.**

Q.5: **What is the length of an arc of a circle of radius 5cm whose central angle is 140º?**

Sol. Here $\ell = ?$, $r = 5\text{cm}$, $\theta = 140°$

$$\theta = 140° = 140 \times \frac{\pi}{180} = 2.44 \text{ rad}$$

By using formula: $\ell = r\theta$

$\ell = r\theta = 5(2.44) = \boxed{12.22\text{cm}}$

Q.6: **Find the length of the arc cut off on a circle of radius 3cm by a central angle of 2 radians.** (IA-2017)

Sol. Here $\ell = ?$, $r = 3\text{cm}$, $\theta = 2$ rad

By using formula: $\ell = r\theta$

$\ell = r\theta = (3)(2) = \boxed{6\text{cm}}$

SOLUTION OF SHORT QUESTIONS

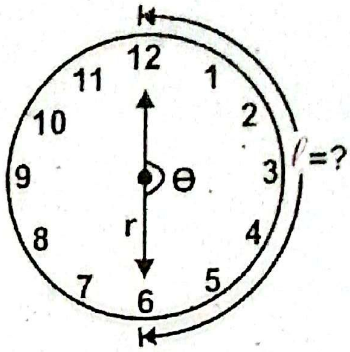
Q.7: Find the radius of a circle when $\ell = 8.4\text{cm}, \theta = 2.8$ rad.

(IA-2016), (IIA-2017)

Sol. Same as Q.3(i) of Ex# 3.1 (see page # 117)

Q.8: If a minute hand of a clock is 10cm long, how for does the tip of the hand moves in 30 minutes? (IA-2018)

Sol. Here $r = 10\text{cm}, \quad \ell = ?$



θ = hand moves in 30 minutes

$$= 180° = 180 \times \frac{\pi}{180} = \pi \text{ rad}$$

By using formula: $\ell = r\theta$

$$\ell = r\theta = (10)\pi = \boxed{31.4\text{cm}}$$

Q.9: Find 'x' if $\tan^2 45° - \cos^2 60° = x\sin 45° \cos 45° \tan 60°$

(IA-2016), (IA-2019)

Sol. $\tan^2 45° - \cos^2 60° = x\sin 45° \cos 45° \tan 60°$

$$(1)^2 - \left(\frac{1}{2}\right)^2 = x\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\left(\sqrt{3}\right)$$

$$1 - \frac{1}{4} = x\left(\frac{2\sqrt{3}}{4}\right)$$

$$\frac{4-1}{4} \times \frac{4}{2\sqrt{3}} = x \quad \Rightarrow \quad \frac{3}{2\sqrt{3}} = x \quad \Rightarrow \quad \boxed{x = \frac{\sqrt{3}}{2}}$$

Q.10: Find 'r' when $\ell = 33\text{cm}, \theta = 6$ radian. (IIA-2020)

Sol. By using formula: $\ell = r\theta$

So $\quad r = \frac{\ell}{\theta} = \frac{33}{6} = \boxed{5.5\text{cm}}$

SOLUTION OF SHORT QUESTIONS

Q.11: Prove that: $2\sin 45^\circ + \frac{1}{2}\operatorname{cosec} 45^\circ = \frac{3}{\sqrt{2}}$

(IIA-2018)

Sol. Same as Q.11(iii) of Ex# 3.2 (see page # 128)

Q.12: Prove that: $\tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ = \frac{13}{3}$

(IA-2016), (IA-2019), (IA-2022)

Sol. $\text{L.H.S.} = \tan^2 30^\circ + \tan^2 45^\circ + \tan^2 60^\circ$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 + (1)^2 + \left(\sqrt{3}\right)^2 = \frac{1}{3} + 1 + 3$$

$$= \frac{1+3+9}{3} = \frac{13}{3} = \text{R.H.S.}$$

Proved.

Q.13: Prove that: $\dfrac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}} = \sqrt{3}$ (IA-2018)

Sol. $$\text{L.H.S.} = \frac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}} = \frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{2} = 3^{1-\frac{1}{2}} = 3^{1/2} = \sqrt{3} = \text{R.H.S.}$$

Proved.

Q.14: Prove that: $\cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ = 0$

(IA-2017), (IIA-2017), (IIA-2018), (IIA-2020)

Sol. Same as Q.13(i) of Ex# 3.2 (see page # 129)

SOLUTION OF SHORT QUESTIONS

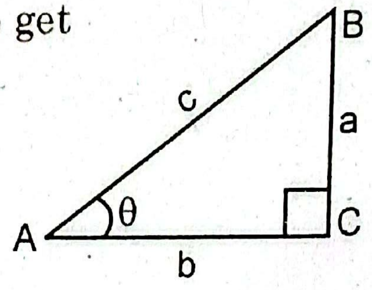
Q.15: Prove that: $\cos 90° - \cos 30° = -2\sin 60° \sin 30°$

Sol. Same as Q.11(iv) of Ex # 3.2 (see page # 128)

Q.16: Prove that: $\sin^2\theta + \cos^2\theta = 1$

Sol. From triangle ABC. By Pythagoras Theorem we know that $a^2 + b^2 = c^2$

Dividing both sides by c^2, we get



$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2}$$

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

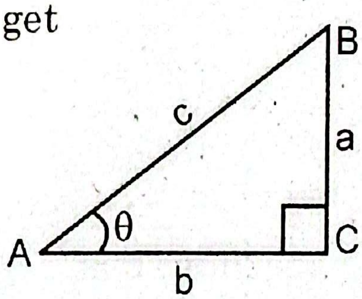
$$\sin^2\theta + \cos^2\theta = 1$$

Proved.

Q.17: Prove that: $1 + \tan^2\theta = \sec^2\theta$

Sol. From triangle ABC. By Pythagoras Theorem we know that $a^2 + b^2 = c^2$.

Dividing both sides by b^2 we get



$$\frac{a^2}{b^2} + \frac{b^2}{b^2} = \frac{c^2}{b^2}$$

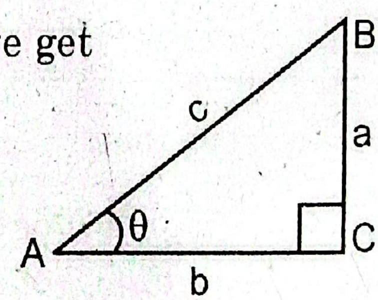
$$\left(\frac{a}{b}\right)^2 + 1 = \left(\frac{c}{b}\right)^2$$

$$\tan^2\theta + 1 = \sec^2\theta \Rightarrow 1 + \tan^2\theta = \sec^2\theta$$ **Proved.**

Q.18: Prove that: $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

Sol. From triangle ABC. By Pythagoras theorem we know that $a^2 + b^2 = c^2$.

Dividing both sides by 'a^2' we get



$$\frac{a^2}{a^2} + \frac{b^2}{a^2} = \frac{c^2}{a^2}$$

$$1 + \left(\frac{b}{a}\right)^2 = \left(\frac{c}{a}\right)^2$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

Proved.

SOLUTION OF SHORT QUESTIONS

Q.19: Prove that: $(1+\sin\theta)(1-\sin\theta)=\dfrac{1}{\sec^2\theta}$

(IIA-2019)

Sol. Same as Q.20 of Ex # 3.3 (see page # 139)

Q.20: Show that: $\cot^4\theta+\cot^2\theta=\text{cosec}^4\theta-\text{cosec}^2\theta$

Sol. L.H.S. $=\cot^4\theta+\cot^2\theta$ (IA-2019)

$=\cot^2\theta\left(\cot^2\theta+1\right)$

$=\cot^2\theta\left(\text{cosec}^2\theta\right)$ $\because \cot^2\theta+1=\text{cosec}^2\theta$

$=\left(\text{cosec}^2\theta-1\right)\left(\text{cosec}^2\theta\right)$ $\because \cot^2\theta=\text{cosec}^2\theta-1$

$=\text{cosec}^4\theta-\text{cosec}^2\theta=\text{R.H.S.}$ **Proved.**

Q.21: Prove that: $\cos\theta+\tan\theta\sin\theta=\sec\theta$ (IA-2021)

Sol. L.H.S. $=\cos\theta+\tan\theta\sin\theta=\cos\theta+\dfrac{\sin\theta}{\cos\theta}.\sin\theta$

$$=\cos\theta+\frac{\sin^2\theta}{\cos\theta}=\frac{\cos^2\theta+\sin^2\theta}{\cos\theta}$$

$$=\frac{1}{\cos\theta}=\sec\theta=\text{R.H.S.}$$ **Proved.**

Q.22: Prove that: $1-2\sin^2\theta=2\cos^2\theta-1$

(IIA-2017), (IA-2018), (IIA-2021)

Sol. Same as Q.1 of Ex# 3.3 (see page # 132)

Q.23: Prove that: $\cos^4\theta-\sin^4\theta=1-2\sin^2\theta$

(IA-2016), (IA-2017), (IIA-2020), (IA-2021)

Sol. Same as Q.2 of Ex# 3.3 (see page # 132)

Q.24: Prove that: $\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta}=2\sec^2\theta$

(IIA-2016), (IIA-2018), (IA-2021), (IIA-2021)

Sol. Same as Q.8 of Ex# 3.3 (see page # 134)